At the end of the year Each Student was given a reading ability test. Its a result as follows.

Group x : 227 176 252 149 16 55 234

Group y. 194 247 92
202 14 165 171 292 271
151 235 147 99

Using the run test. Test whether the quality of reading ability of two groups.

H_0: The quality of reading ability of two group good.

Combine the two samples x and y
The two samples are Combined together in ascending.

14	16	55	92	99	147	149	151	165
Y	X	X	X	Y	Y	X	Y	Y
171	176	194	202	227	234	235	247	
Y	X	X	Y	X	X	Y	X	
252	271	292						
X	Y	Y						

Y | X X X | Y Y | X | Y Y Y | X X | Y | X X | Y | X X | Y Y

No. of runs : 11

m : 10

n : 10

$$E(r) = \frac{2mn}{m+n} + 1$$

$$= \frac{2(10)(10)}{10+10} + 1$$

$$= \frac{200}{20} + 1$$

$$= 10 + 1 = 11$$

$$V(r) = \frac{2mn(2mn - m - n)}{(m+n)^2(m+n-1)}$$

$$= \frac{2(10)(10)(2(10)(10) - 10 - 10)}{(10+10)^2(10+10-1)}$$

$$= \frac{(200)(200-10-10)}{(400)(19)}$$

$$= \frac{(200)(180)}{7600} = \frac{36000}{7600} = 4.7368$$

$$|Z| = \left| \frac{r - E(r)}{\sqrt{V(r)}} \right| = \left| \frac{11 - 11}{\sqrt{4.7368}} \right| = \frac{0}{2.1764}$$

The Value at 5% level of Significance is 1.96

$\therefore$ The Calculated value is less than table value

$\therefore$ H_0 is accepted.

Sign Test for Single Sample:

1. The following are the measurement of bringing strength of a certain kind of 20 cotton ribbon in bundle.

163 165 160 189 161 171 158 151 169 162

163 139 172 165 148 166 172 163 187 173

Using sign Test to test the null hypothesis $\mu = 160$ against $\mu \geq 160$ at 5% l.o.s

Sln:

$H_0 : \mu = 160$

$n = 20 \; (n < 30)$

$$p' = \left(\frac{1}{2}\right)^n \sum_{x=0}^{r} nC_x .$$

when r is the no of '−' sign

+, +, 0, +, +, +, −, −, +, +, +, −, +, +, −, +, +, +, +, +

No. of + Sign : 15

No. of - sign = 4

No. of zero = 1

$n = 15 + 4$

$n = 19$

$$p' = \left(\frac{1}{2}\right)^{19} \sum_{x=0}^{4} 19^{c}_{x}$$

$$= \left(\tfrac{1}{2}\right)^{19} \left[19^{c}_{0} + 19^{c}_{1} + 19^{c}_{2} + 19^{c}_{3} + 19^{c}_{4}\right]$$

$$= \frac{1}{524288} \left[1 + 19 + 171 + 969 + 3876\right]$$

$$= \frac{5036}{524288}$$

$p' = 0.0096$

2. 20 observations are given below:-

93, 97, 98, 88, 105, 82, 107, 86, 103, 113, 107,

112, 90, 99, 93, 100, 103, 96, 104, 101

Test the whether the median is 99 or not by using sign test $|z| = 0.5$

Sln:-

$H_0 : \mu = 99$

$n = 20 \ (n < 30)$

$$p' = \left(\frac{1}{2}\right)^{n} \sum_{x=0}^{r} n^{c}_{x}$$

where 'r' is the no. of '-' sign

-, -, -, -, +, -, +, -, +, +, +, +, -, 0, -, +, +, -,

No. of + Sign : 10 $\therefore n = 10 + 9$
No. of - Sign : 9 $= 19$
No. of zero : 1

$$p' = \left(\frac{1}{2}\right)^{19} \sum_{x=0}^{9} 19C_x$$

$$= (1/2)^{19} [19C_0 + 19C_1 + 19C_2 + 19C_3 + 19C_4 + 19C_5 + 19C_6 + 19C_7 + 19C_8 + 19C_9]$$

$$= \frac{1}{\cancel{1048576}\ 524288} [1 + 19 + 171 + 969 + 3876 + 11628 + 27132 + 50388 + 75582 + 92378]$$

$$= \frac{262144}{\cancel{1048576}\ 524288} = 0.5$$

3. 30 observations are given below.

30, 20, 22, 14, 21, 49, 64, 58, 44, 39, 45, 55, 81, 92, 64, 55, 11, 9, 8, 54, 61, 39, 28, 59, 76, 75, 76, 49, 49, 63

Test whether the median 67 or not by Using sign test $\boxed{|z| = 2.9212}$ Ans

Sln :

$H_0 : \mu = 67$

$n = 30 \quad (n > 30)$

$$p' = \left(\frac{1}{2}\right)^n \sum_{x=0}^{r} nC_x$$

Where 'r' is the no. of '+' sign

−, −, −, −, −, +, −, −, −, −, −, −, +, +, −, −,
−, −, −, −, −, −, −, −, +, +, +, −, +, −.

No. of + Sign: 7

No. of − Sign: 23

$\therefore n = 30$

$r = '+'$

$p' \neq \left(\frac{1}{2}\right) \sum_{x=0}$

$$Z = \frac{r - n/2}{\sqrt{n/4}}$$

$$Z = \left| \frac{7 - 30/2}{\sqrt{30/4}} \right|$$

$$= \left| -\frac{8}{\sqrt{7.5}} \right|$$

$$= \left| \frac{-8}{2.7386} \right|$$

$$= 2.9212.$$

Sign Test for Two random Samples:-

1) The two random Samples are drawn from the two population, the Sample Values are as follows. (12,9) (21,20) (22,22) (32,35) (20,26) (7,8) (10,7) (15,20) (16,26)
Test whether the 2 population are equal

Sol:

H_0: The two populations are equal.

$n=9 \quad (n<30)$

$$p' = \left(\frac{1}{2}\right)^n \sum_{x=0}^{r} n_{C_x}$$

where 'r' is the no. of '−' sign

+ + 0 − − − − + − −

No. of + sign = 3

No. of − sign = 5 $\qquad n=8$

No. of zero = 1

$$p' = \left(\frac{1}{2}\right)^8 \sum_{x=0}^{5} 8_{C_x}$$

$$= \frac{1}{256}\left[8_{C_0} + 8_{C_1} + 8_{C_2} + 8_{C_3} + 8_{C_4} + 8_{C_5}\right]$$

$$= \frac{1}{256}\left[1+8+28+56+70+56\right]$$

$$= \frac{219}{256}$$

$= 0.8555$

The calculated value = 0.8555 is less than table value

H_0 is accepted

2. Using the Sign Test to test whether the two populations are equal with the help of following Values.

(2,3) (7,8) (10,20) (20,8) (16,17) (21,15) (26,20)

(30,32) (25,28) (26,20) (17,15) (12,12) (13,14)

(15,20) (30,20) (36,33) (28,30) (40,52) (30,35)

(28,25) (26,26) (30,36) (32,38) (36,30) (25,30)

(26,20) (9,12) (11,25) (7,5) (6,9) (16,19) (26,23)

(28,31) (32,38) (40,45).

Sol:

H_0: The two populations are equal

$\boxed{n = 35}$ $(n > 35)$

$$z = \frac{r - n/2}{\sqrt{n/4}}$$

Where 'r' is the no. of '+' sign.

− − − + − + + − − + + 0 − − + + − − −

+ 0 − − + − + − − + − − + − − −

No. of + Sign = 12 $\qquad$ $n = 12 + 21$

No. of − Sign = 21 $\qquad$ $= 33$

No. of zero = 2

$$|z| = \left| \frac{12 - 33/2}{\sqrt{33/4}} \right|$$

$$= \left| \frac{-4.5}{\sqrt{8.25}} \right| = \left| \frac{-4.5}{2.8723} \right|$$

$$= 1.5667$$

The Calculated Value 1.56667 is less than table Value.

$\therefore H_0$ is accepted.

3. Use the sign test to see if there is a difference between the no. of days required to Collect an account receivable before and after a new Collection policy and you may assume. 0.05 l.o.s

Before : 33 36 41 32 39 47 34 29 32

After : 35 29 38 34 37 47 36 32 30

Before : 34 40 42 33 36 27

After : 34 41 38 37 35 28

Sln :

H_0: There is no difference between the no. of days required to Collect an account.

$n = 15$ $(n < 30)$

$$= p' = \left(\frac{1}{2}\right)^n \sum_{x=0}^{r} n_{C_x}$$

where r is no. of '-' sign

$- + + - + 0 - - + 0 - + - + -$

no. of + Sign = 6

No. of - Sign = 7

No. of zero = 2

$n = 6 + 7$

$= 13$

$$p' = \left(\frac{1}{2}\right)^{13} \sum_{x=0}^{7} 13C_x$$

$$= \frac{1}{8192}\left[13C_0 + 13C_1 + 13C_2 + 13C_3 + 13C_4 + 13C_5 + 13C_6 + 13C_7\right]$$

$$= \frac{1}{8192}\left[1 + 13 + 78 + 286 + 715 + 1287 + 1716 + 1716\right]$$

$$= 5812/8192$$

$$p' = 0.7095$$

The Calculated Value $= 0.7095$ is less than table Value $= 1.96$

$\therefore H_0$ is accepted.

Median Test

1. The following data Using the lifetime of bulbs of two different brand. A Sample of 7 bulbs from brand 1 and a Sample of 8 bulbs from brand 2 is Selected

Brand$_1$ (x_1): 80 100 90 110 125 180 70

Brand$_2$ (y): 100 120 80 140 130 160 115

Test whether the median lifetime of bulb of 2 brands are equal at 5% l.o.s

Sol :

H_0: The median lifetime of bulbs of 2 brands are equal.

· To Combine the given two Samples and arrange the Values in ascending Order.

· 70, 80, 80, 90, 100, 100, 110, 115, 120, 120, 125, 130, 130, 140, 160.

$$\text{Median} = \text{Size of } \left(\frac{n+1}{2}\right) \text{Value}$$

$$= \text{Size of } \left(\frac{15+1}{2}\right) \text{Value}$$

$$= \text{Size of } 8 \text{ Value}$$

2.

Median $= 115$

	X	Y	Total
No. of < 115	5 a	2 b	7
No. of ≥ 115	2 c	6 d	8
Total	7	8	15

$$\chi^2 = \frac{N(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)} \sim \chi^2_{(r-1)(c-1)}$$

$$= \frac{15(5\times 6 - 2\times 2)^2}{7\times 8\times 7\times 8} \quad \chi^2_{(1)}$$

$$= \frac{15(676)}{3136} = \frac{10140}{3136}$$

$$= 3.2334.$$

Table Value $= 3.841$.

Since Our calculated Value is ~~greater~~ less than table Value. Accepting Our H_0

2. The two Sample Sizes 18 and 23 are taken from the populations. To test whether the samples are drawn from the same population by Using Median test

Sample 1: 220 224 248 230 245 315 2[illegible]
247 254 251 225 235 255 225
207 206 268 256

Sample 2: 206 214 236 225 243 239 211
232 250 229 206 207 226 258
258 243 222 227 225 205 228
259 237

Solution:

H_0: The ~~two~~ samples are drawn from same population.

To combine the given two samples and arrange the values in ascending order.

205, 206, 206, 206, 207, 207, 211, 214, 220, 222, 224, 225, 225, 225, 225, 226, 227, 228, 229, 230, 232, 235, 236, 237, 239, 243, 243, 245, 247, 248, 250, 250, 251, 254, 255, 255, 256, 258, 259, 268, 315

Median = Size of $\left(\frac{n+1}{2}\right)$ Value

= Size of $\left(\frac{41+2}{2}\right)$ Value

Size of 21 Value

Median = 232.

	X	Y	Total
No. of ob < 232	7 a	13 b	20
No. of ≥ 232	11 c	10 d	21
Total	18	23	41

$$\chi^2 = \frac{N(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)} \sim \chi^2_{(r-1)(c-1)}$$

$$= \frac{41(7\times 10 - 13\times 11)^2}{(20\times 21\times 18\times 23)}$$

$$= \frac{41(5329)}{173880} = \frac{218489}{173880}$$

$$= 1.2566.$$

∴ Table Value = ~~3.8466~~ 3.841

Since Our Calculated Value is less than table Value ∴ so we accept Our H_0.

Manual Mann whitney U-Test

1. In the following data was giv to 7 Students from rural area & 25 students from Urban area with respect to the interesting our hig education. Choose Mann whitney U-Tes to test the hypothesis that these Students in both of the area are equally interested to Continue our higher education.

Score of rural students } : 7.3 5.6 6.3 9.0 4.2 10.6 2.7

Score of Urban Students } : 7.6 6.4 2.4 5.0 5.6 2.4 6.1 10.6 5.6 1.0 6. 10.6 6.7 3.6 3.2 9.9 8. 6.7 1.8 5.9 7.9 9.9 10. 2.0 4.0

Sln :

H_0: The both area of the student are equally interested to Continue their higher education.

x	y	x	y	R_x	R_y	$R_1 = \Sigma R_x$
7.3	7.6	2.7	1.0	6	1	
5.6	6.4	4.2	1.8	10	2	
6.3	2.4	5.6	2.0	13	3	
9.0	5.0	6.3	2.4	18	4.5	
4.2	5.6	7.3	2.4	22	4.5	
10.6	2.4	9.0	3.2	26	7	
2.7	6.1	10.6	3.6	30.5	8	
	10.6		4.0	125.5	9	
	5.6		5.0		11	
	1.0		5.6		13	
	6.0		5.6		13	
	10.6		5.9		15	
	6.7		6.0		16	
	3.6		6.1		17	
	3.2		6.4		19	
	9.9		6.7		20.5	
	8.3		6.7		20.5	
	6.7		7.6		23	
	1.8		7.9		24	
	5.9		8.3		25	
	7.9		9.9		27.5	2, 3, 2, 2, 3
	9.9		9.9		27.5	
	10.6		10.6		30.5	
	2.0		10.6		30.5	
	4.0		10.6		30.5	
					402.5	

$R_1 = \Sigma R x$

$R_1 = 125 \cdot 5$

$R_2 = \Sigma R y = 402 \cdot 5$

$$U_{12} = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1$$

$$= (7 \times 25) + \frac{7(8)}{2} - 125 \cdot 5$$

$$= 175 + 28 - 125 \cdot 5$$

$$U_{12} = 77 \cdot 5$$

$$U_{21} = n_1 n_2 - U_{12}$$

$$= (7 \times 25) - 77 \cdot 5$$

$$= 175 - 77 \cdot 5$$

$$= 97 \cdot 5$$

$$U = Min(U_{12}, U_{21})$$

$$= Min(77 \cdot 5, 97 \cdot 5)$$

$$U = 77 \cdot 5$$

No. of times repeated

2, 3, 2, 2, 4

$(t_1 = 2,\ t_2 = 3,\ t_3 = 2,\ t_4 = 2,\ t_5 = 4)$

$$T_1 = \frac{t_1^3 - t_1}{12} = \frac{2^3 - 2}{12} = \frac{8-2}{12} = \frac{1}{2}$$

$$T_2 = \frac{t_2^3 - t_2}{12} = \frac{3^3 - 3}{12} = \frac{27 - 3}{12} = 2$$

$$T_3 = \frac{t_3^3 - t_3}{12} = \frac{2^3 - 2}{12} = \frac{1}{2}$$

$$T_4 = \frac{t_4^3 - t_4}{12} = \frac{(2)^3 - 2}{12} = \frac{1}{2}.$$

$$T_5 = \frac{t_5^3 - t_5}{12} = \frac{4^3 - 4}{12} = \frac{64 - 4}{12} = \frac{60}{12} = 5.$$

$$\Sigma T_i = T_1 + T_2 + T_3 + T_4 + T_5$$

$$= 8.5$$

$$Z = \frac{U - \frac{n_1 n_2}{2}}{\sqrt{\frac{n_1 n_2 (N+1)}{12} - \frac{n_1 n_2}{N(N-1)} \Sigma T_i}}$$

$$= \frac{77.5 - \frac{(7)(25)}{2}}{\sqrt{\frac{(7)(25)(33)}{12} - \frac{(7)(25)}{32(32-1)} 8.5}}$$

185

$$= \frac{77.5 - 87.5}{\sqrt{\frac{5775}{12} - \frac{\cancel{1487.5}\ 175}{32(31)}}} = \frac{-10}{\sqrt{481.25 - 0.1}}$$

$$= \frac{-10}{\sqrt{481.25 - 1.4995}}$$

$$= \frac{-10}{\sqrt{479.7505}}$$

$$|z| = \frac{-10}{21.9032} = 0.4566.$$

T.V = 1.96.

Since Our Calculated Value less than table Value. 1.96. So we accepting Our H_0.

2. A Survey is Conducted to test the difference between two alternat methods of teaching. A Sample o 20 Students is Selected at rand Two groups of 10 Students each of equal ability are formed, and thought by different methods. A Standarised test is then given to both the groups and the following marks (out of 100) are Scored by the 10 students in each group.

Group A: 40 45 48 46 52 58 72 85
67 73

Group B: 42 68 45 64 85 78 87 62 84
90

Using U-test at 5% l.o.s, to test the significance of difference between the performance of two groups.

3. A large Corporate hospital hires most of its doctors from two major Universities. Over the last year hospital has been conducting test for the newly recruited doctors to determine which University educates better. Based on the following scores, help the human resource department of the hospital to decide whether the Universities differ in quality. Use Man whitney U test and you may assume 5% l.o.s

Uni A: 99 83 89 64 98 85 61 79 91 87
88

Uni B: 96 90 97 94 86 95 68 78 93 56
76 84